Pizzablatt 3 / Aufgabe 1

a) $$W(x_1, \dots x_d) = \frac{1}{1 - x_1 - \dots - x_d - x_1x_2 - x_1x_3 - \dots \dots - x_1x_2 \cdots x_d} = \frac{1}{2 - \prod_{i=1}^{d}(1+x_i)}$$

b) $$M := \sum_{n_1 \dots n_k \geq 1} \min\{n_1, \dots n_k\}\, t_1^{n_1} \dots t_k^{n_k}$$

Dann ist $(1 - t_1 \dots t_k) M =$

$$= \sum_{n_1 \dots n_k \geq 1} \left(\min\{n_1 \dots n_k\} - \min\{n_1 - 1, \dots n_k - 1\} \right) t_1^{n_1} \dots t_k^{n_k}$$

$$= \sum_{n_1 \dots n_k \geq 1} t_1^{n_1} \dots t_k^{n_k}$$

Also: $$\frac{(1 - t_1 \dots t_k)}{t_1 \dots t_k} M = \sum_{n_1 \dots n_k \geq 0} t_1^{n_1} \dots t_k^{n_k} = \frac{1}{\prod_{i=1}^{k}(1 - t_i)}$$

3.4) 2 Vertauschungsmatrizen

$$\begin{pmatrix} 1 & & & \\ & 1 & & \\ & & 1 & \\ & & & 1 \end{pmatrix} \quad \text{und} \quad \begin{pmatrix} & 1 & & \\ 1 & & & \\ & & & 1 \\ & & 1 & \end{pmatrix}$$

char. Polynom $\det(1 - tM)$

$$(1-t)^4 \qquad\qquad (1-t^2)^2$$

Hilbert - Poincare - Reihe:

$$\frac{1}{2}\left(\frac{1}{(1-t)^4} + \frac{1}{(1-t^2)^2}\right) = \frac{1+t^2}{(1-t)^4(1+t)^2}$$

$$= 1 + 2t + 6t^2 + 10t^3 + 19t^4 + 28t^5 + 44t^6 + \dots$$

Blatt 3, Aufgabe 5

a) $\begin{array}{ccc}1&-&2\\ |&&|\\ 4&-&3\end{array}$ $()$ $\qquad$ $\begin{array}{ccc}1&-&4\\ |&&|\\ 2&-&3\end{array}$ (24)

$\begin{array}{ccc}4&-&1\\ |&&|\\ 3&-&2\end{array}$ (1234) $\qquad$ $\begin{array}{ccc}4&-&3\\ |&&|\\ 1&-&2\end{array}$ $(14)(32)$

$\begin{array}{ccc}3&-&4\\ |&&|\\ 2&-&1\end{array}$ $(13)(24)$ $\qquad$ $\begin{array}{ccc}3&-&2\\ |&&|\\ 4&-&1\end{array}$ (13)

$\begin{array}{ccc}2&-&3\\ |&&|\\ 1&-&4\end{array}$ (1432) $\qquad$ $\begin{array}{ccc}2&-&1\\ |&&|\\ 3&-&4\end{array}$ $(12)(34)$

$p_{1111} = x_1 + x_2 + x_3 + x_4$

$p_{211} = p_2 p_1^2$

$p_{1111} = p_1^4$

$p_2 = x_1^2 + x_2^2$

$$Z_G = \frac{1}{8}\left(p_{1111} + 2p_{211} + 3p_{22} + 2p_4\right)$$

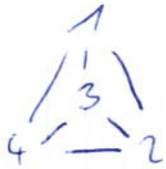
Anzahl Möglichkeiten, das Quadrat zu färben:

Setze ein: $x_1 = 1 = \ldots = x_K$, $x_{K+1} = 0 = x_{K+2} = \ldots$

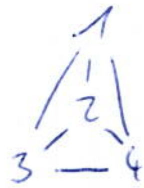
$$\rightsquigarrow \frac{1}{8}\left(k^4 + 2k^3 + 3k^2 + 2k\right)$$

b)

$()$



(234)



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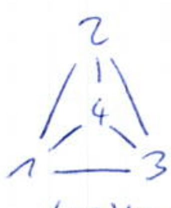
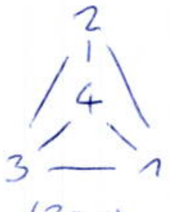


$(12)(34)$



(321)

(142)

$$Z_G = \frac{1}{12}\left(p_{1111} + 8p_{31} + 3p_{22}\right) = 2m_{1111} + m_{211} + m_{22} + m_{31} + m_4$$

$\rightsquigarrow$ k Einsen einsetzen

$$\frac{1}{12}\left(k^4 + 8k^2 + 3k^2\right) = \frac{k^4 + 11k^2}{12}$$